

Direct CP violation for $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ decay in QCD factorization *

Gang Lü

*College of Science,
Henan University of Technology,
Zhengzhou 450001, China*

Bao-He Yuan

*North China University of Water Resources and Electric Power,
zhengzhou 450011, China*

Ke-Wei Wei[†]

*Institute of High Energy Physics,
Chinese Academy of Sciences,
Beijing 100049, China*

(Dated: November 16, 2010)

Abstract

In the framework of QCD factorization, based on the first order of isospin violation, we study direct CP violation in the decay of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$ including the effect of $\rho - \omega$ mixing. We find that the CP violating asymmetry is large via $\rho - \omega$ mixing mechanism when the invariant mass of the $\pi^+ \pi^-$ pair is in the vicinity of the ω resonance. For the decay of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$, the maximum CP violating asymmetries can reach about 46%. We also discuss the possibility to observe the predicted CP violating asymmetries at the LHC.

PACS numbers: 11.30.Er, 12.39.-x, 13.20.He, 12.15.Hh

*ganglv@haut.edu.cn

[†]Electronic address: weikw@ihep.ac.cn

I. INTRODUCTION

CP violating asymmetry is one of the most important areas in the decays of bottom hadrons. In the standard model (SM), a non-zero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is responsible for CP violating phenomena. In recent years CP violation in several B decays such as $B^0 \rightarrow J/\psi K_S^0$ and $B^0 \rightarrow K^+\pi^-$ has indeed been found in experiments [1, 2]. Due to its much higher statistics, the Large Hadron Collider (LHC) will provide a new opportunity to search for more CP violation signals.

Direct CP violating asymmetries in b -hadron decays occur through the interference of at least two amplitudes with the weak phase difference ϕ and the strong phase difference δ . The weak phase difference is determined by the CKM matrix while the strong phase is usually difficult to control. In order to have a large CP violating asymmetries signal, we have to apply some phenomenological mechanism to obtain a large δ . It has been shown that the charge symmetry violating mixing between ρ^0 and ω can be used to obtain a large strong phase difference which is required for large CP violating asymmetries. Furthermore, it has been shown that the measurement of the CP violating asymmetries can be used to remove the $\text{mod}(\pi)$ ambiguity in the determination of the CP violating phase angle α [3–7].

Naive factorization approximation has been shown to be the leading order result in the framework of QCD factorization when the radiative QCD corrections of order $O(\alpha_s(m_b))$ (m_b is the b -quark mass) and the $O(1/m_b)$ corrections in the heavy quark effective theory are neglected [8]. In naive factorization scheme, the hadronic matrix elements of four-quark operators are assumed to be saturated by vacuum intermediate states. Since the bottom hadrons are very heavy, their hadronic decays are energetic. Hence the quark pair generated by one current in the weak Hamiltonian moves very fast away from the weak interaction point. Therefore, by the time this quark pair hadronizes into a meson, it is already far away from other quarks and is unlikely to interact with the remaining quarks. This quark pair is factorized out and generates a meson [9, 10]. This approximation can only estimate the CP violation order neglecting QCD correction. Furthermore, as pointed out in previous studies [5–7], in order to taken into account the nonfactorizable contributions, an effective parameter, N_c , is introduced. The deviation of the value of N_c from the color number, 3, measures the nonfactorizable effects in the naive factorization scheme. Obviously, N_c should depend on the hadronization dynamics of different decay channels. In this scheme, CP violation depends strongly on N_c values, which makes the results uncertainties.

In the heavy quark limit, QCD factorization [8] includes nonfactorization strong interaction correction, and the decay amplitudes can be calculated at leading power in $\frac{\Lambda_{QCD}}{m_b}$ and at next-to-leading order in α_s , which can be expressed in terms of form factors and meson light-cone distribution amplitudes. One can take into account the nonfactorizable and chirally enhanced hard-scattering spectator and annihilation contributions which appear at order $O(\alpha_s(m_b))$ and $O(1/m_b)$, respectively. In this work we adopt the QCD factorization scheme including order- α_s correction to compute CP violating asymmetry of the decay $\bar{B}_s^0 \rightarrow K^0\pi^+\pi^-$ via the $\rho - \omega$ mixing mechanism. As will be shown later, the CP violating asymmetries in this decay channel could be large and may be observed in the LHC experiments.

The remainder of this paper is organized as follows. In Sec. II, we present the form

of the effective Hamiltonian and the general form of QCD factorization. In Sec. III, we give the formalism for CP violating asymmetries in $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ decay. In Sec. IV, we calculate the branching ratio for decay process of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$ via $\rho - \omega$ mixing. We briefly discuss the input parameters in Sec. V. The numerical results are given in Sec. VI. In Sec. VII we discuss the possibility to observe the predicted CP violating asymmetries at the LHC. Summary and conclusions are included in Sec. VIII.

II. THE EFFECTIVE HAMILTONIAN

With the operator product expansion [11], the effective Hamiltonian in bottom hadron decays is

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{G_F}{\sqrt{2}} \left[\sum_{p=u,c} \sum_{q=d,s} V_{pb} V_{pq}^* (c_1 O_1^p + c_2 O_2^p) \right. \\ & \left. + \sum_{i=3}^{10} c_i O_i + c_{7\gamma} O_{7\gamma} + c_{8g} O_{8g} \right] + H.c., \end{aligned} \quad (1)$$

where c_i ($i = 1, \dots, 10, 7\gamma, 8g$) are the Wilson coefficients, V_{pb} , V_{pq} are the CKM matrix elements. The operators O_i have the following form:

$$\begin{aligned} O_1^p &= \bar{p} \gamma_\mu (1 - \gamma_5) b \bar{q} \gamma^\mu (1 - \gamma_5) p, & O_2^p &= \bar{p}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{q}_\beta \gamma^\mu (1 - \gamma_5) p_\alpha, \\ O_3 &= \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', & O_4 &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \\ O_5 &= \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', & O_6 &= \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_7 &= \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 + \gamma_5) q', & O_8 &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 + \gamma_5) q'_\alpha, \\ O_9 &= \frac{3}{2} \bar{q} \gamma_\mu (1 - \gamma_5) b \sum_{q'} e_{q'} \bar{q}' \gamma^\mu (1 - \gamma_5) q', & O_{10} &= \frac{3}{2} \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta \sum_{q'} e_{q'} \bar{q}'_\beta \gamma^\mu (1 - \gamma_5) q'_\alpha, \\ O_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b, & O_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b, \end{aligned} \quad (2)$$

where α and β are color indices, O_1^p and O_2^p are the tree operators, $O_3 - O_6$ are QCD penguin operators which are isosinglets, $O_7 - O_{10}$ arise from electroweak penguin operators which have both isospin 0 and 1 components. $O_{7\gamma}$ and O_{8g} are the electromagnetic and chromomagnetic dipole operators. $e_{q'}$ are the electric charges of the quarks and $q' = u, d, s, c, b$ is implied.

The Wilson coefficients can be calculated at a high scale M_W and then evolved to scale m_b using renormalization group equation. In QCD factorization, We consider weak decay $B_s \rightarrow M_1 M_2$ (M_1, M_2 refer to K^0 and ρ^0 mesons, respectively) in the heavy-quark limit. Up to power corrections of order Λ_{QCD}/m_b , the transition matrix element of an operator

\mathcal{O}_i in the weak effective Hamiltonian is given by[8]

$$\begin{aligned} \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle &= \sum_j F_j^{B \rightarrow M_1}(m_2^2) \int_0^1 du T_{ij}^I(u) \Phi_{M_2}(u) \\ &+ (M_1 \leftrightarrow M_2) \\ &+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u) \\ &\text{if } M_1 \text{ and } M_2 \text{ are both light,} \end{aligned} \quad (3)$$

Here $F_j^{B \rightarrow M_{1,2}}(m_{2,1}^2)$ denotes a $B \rightarrow M_{1,2}$ form factor, and $\Phi_X(u)$ is the light-cone distribution amplitude for the quark-antiquark Fock state of meson X . $T_{ij}^I(u)$ and $T_i^{II}(\xi, u, v)$ are hard-scattering functions, which are perturbatively calculable. The hard-scattering kernels and light-cone distribution amplitudes (LCDA) depend on a factorization scale and scheme, which is suppressed in the notation of (3). Finally, $m_{1,2}$ denote the light meson masses.

We match the effective weak Hamiltonian onto a transition operator, the matrix element is given by $(\lambda_p^{(D)} = V_{pb} V_{pD}^*$ with $D = d$ or s)

$$\langle M'_1 M'_2 | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \sum_{p=u,c} \lambda_p^{(D)} \langle M'_1 M'_2 | \mathcal{T}_A^p + \mathcal{T}_B^p | \bar{B} \rangle. \quad (4)$$

Using the unitarity relation

$$\lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0 \quad (5)$$

we can get

$$\begin{aligned} \sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_A^p &= \sum_{p=u,c} \lambda_p^{(D)} \left[\delta_{pu} \alpha_1(M_1 M_2) A([\bar{q}_s u][\bar{u} D]) + \delta_{pu} \alpha_2(M_1 M_2) A([\bar{q}_s D][\bar{u} u]) \right] \\ &+ \lambda_u^{(D)} \left[(\alpha_4^u(M_1 M_2) - \alpha_4^c(M_1 M_2)) \sum_q A([\bar{q}_s q][\bar{q} D]) + (\alpha_{4,\text{EW}}^u(M_1 M_2) - \alpha_{4,\text{EW}}^c(M_1 M_2)) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]) \right. \\ &- \lambda_t^{(D)} \left[\alpha_3^c(M_1 M_2) \sum_q A([\bar{q}_s D][\bar{q} q]) + \alpha_4^c(M_1 M_2) \sum_q A([\bar{q}_s q][\bar{q} D]) + \alpha_{3,\text{EW}}^c(M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s D][\bar{q} q]) \right. \\ &\left. \left. + \alpha_{4,\text{EW}}^c(M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]) \right] \right] \end{aligned}$$

where the sums extend over $q = u, d, s$, and \bar{q}_s denotes the spectator antiquark. The operators $A([\bar{q}_{M_1} q_{M_1}][\bar{q}_{M_2} q_{M_2}])$ also contain an implicit sum over $q_s = u, d, s$ to cover all possible B -meson initial states.

Next we need change the annihilation part

$$\begin{aligned}
\sum_{p=u,c} \lambda_p^{(D)} \mathcal{T}_B^p &= \sum_{p=u,c} \lambda_p^{(D)} \\
&\times \left[\delta_{pu} b_1(M_1 M_2) \sum_{q'} B([\bar{u}q'][\bar{q}'u][\bar{D}b]) \right. \\
&+ \delta_{pu} b_2(M_1 M_2) \sum_{q'} B([\bar{u}q'][\bar{q}'D][\bar{u}b]) \left. \right] \\
&- \lambda_t^{(D)} \left[b_3(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) \right. \\
&+ b_4(M_1 M_2) \sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \\
&+ b_{3,\text{EW}}(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q'][\bar{q}'D][\bar{q}b]) \\
&+ b_{4,\text{EW}}(M_1 M_2) \sum_{q,q'} \frac{3}{2} e_q B([\bar{q}q'][\bar{q}'q][\bar{D}b]) \left. \right]
\end{aligned} \tag{7}$$

where b_i , $b_{i,\text{EW}}$ and B are given by following. The coefficients of the flavor operators α_i^p can be expressed in terms of the coefficients a_i^p defined in [8] as follows:

$$\begin{aligned}
\alpha_1(M_1 M_2) &= a_1(M_1 M_2), \\
\alpha_2(M_1 M_2) &= a_2(M_1 M_2), \\
\alpha_3^p(M_1 M_2) &= \begin{cases} a_3^p(M_1 M_2) + a_5^p(M_1 M_2); \\ \text{if } M_1 M_2 = PV, \end{cases} \\
\alpha_4^p(M_1 M_2) &= \begin{cases} a_4^p(M_1 M_2) + r_\chi^{M_2} a_6^p(M_1 M_2); \\ \text{if } M_1 M_2 = PV, \end{cases} \\
\alpha_{3,\text{EW}}^p(M_1 M_2) &= \begin{cases} a_9^p(M_1 M_2) + a_7^p(M_1 M_2); \\ \text{if } M_1 M_2 = PV, \end{cases} \\
\alpha_{4,\text{EW}}^p(M_1 M_2) &= \begin{cases} a_{10}^p(M_1 M_2) + r_\chi^{M_2} a_8^p(M_1 M_2); \\ \text{if } M_1 M_2 = PV, \end{cases}
\end{aligned} \tag{8}$$

For pseudoscalar (P) meson M_1 , the ratios $r_\chi^{M_1}$ are defined as

$$r_\chi^{M_1}(\mu) = \frac{2m_{M_1}^2}{m_b(\mu)(m_q + m_s)(\mu)}, \tag{9}$$

All quark masses are running masses defined in the $\overline{\text{MS}}$ scheme, and m_q denotes the average of the up and down quark masses. For vector (V) meson M_2 we have

$$r_\chi^{M_2}(\mu) = \frac{2m_V}{m_b(\mu)} \frac{f_V^\perp(\mu)}{f_V}, \tag{10}$$

where the scale-dependent transverse decay constant f_V^\perp is defined as

$$\langle V(p, \varepsilon^*) | \bar{q} \sigma_{\mu\nu} q' | 0 \rangle = f_V^\perp (p_\mu \varepsilon_\nu^* - p_\nu \varepsilon_\mu^*). \quad (11)$$

Note that all the terms proportional to $r_\chi^{M_2}$ are formally suppressed by one power of Λ_{QCD}/m_b in the heavy-quark limit.

The general form of the coefficients a_i^p at next-to-leading order in α_s is

$$\begin{aligned} a_i^p(M_1 M_2) = & \left(C_i + \frac{C_{i\pm 1}}{N_c} \right) N_i(M_2) \\ & + \frac{C_{i\pm 1}}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] \\ & + P_i^p(M_2), \end{aligned} \quad (12)$$

where N_c is the number of colors, the upper (lower) signs apply when i is odd (even). It is understood that the superscript ‘ p ’ is to be omitted for $i = 1, 2$. The quantities $V_i(M_2)$ account for one-loop vertex corrections, $H_i(M_1 M_2)$ for hard spectator interactions, and $P_i^p(M_1 M_2)$ for penguin contractions. The $N_i(M_2)$ and C_F are given by

$$N_i(M_2) = \begin{cases} 0; & i = 6, 8 \text{ and } M_2 = V, \\ 1; & \text{all other cases.} \end{cases} \quad (13)$$

$$C_F = \frac{N_c^2 - 1}{2N_c}. \quad (14)$$

The vertex corrections are given by[8]

$$V_i(M_2) = \begin{cases} \int_0^1 dx \Phi_{M_2}(x) \left[12 \ln \frac{m_b}{\mu} - 18 + g(x) \right] \\ (i = 1 - 4, 9, 10), \\ \int_0^1 dx \Phi_{M_2}(x) \left[-12 \ln \frac{m_b}{\mu} + 6 - g(1-x) \right] \\ (i = 5, 7), \\ \int_0^1 dx \Phi_{m_2}(x) \left[-6 + h(x) \right] \\ (i = 6, 8), \end{cases} \quad (15)$$

with

$$\begin{aligned} g(x) = & 3 \left(\frac{1-2x}{1-x} \ln x - i\pi \right) + \left[2 \text{Li}_2(x) - \ln^2 x \right. \\ & \left. + \frac{2 \ln x}{1-x} - (3 + 2i\pi) \ln x - (x \leftrightarrow 1-x) \right], \end{aligned} \quad (16)$$

$$h(x) = 2 \text{Li}_2(x) - \ln^2 x - (1 + 2\pi i) \ln x - (x \leftrightarrow 1-x). \quad (17)$$

The constants $-18, 6, -6$ are scheme dependent and correspond to using the NDR scheme for γ_5 . The light-cone distribution amplitude (LCDA) Φ_{M_2} is the leading-twist amplitude

of M_2 , whereas Φ_{m_2} is the twist-3 amplitude. LCDA for pseudoscalar and vector mesons of twist-2 are

$$\begin{aligned}\Phi_P(x, \mu) &= 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^P(\mu) C_n^{3/2}(2x-1) \right], \\ \Phi_{\parallel}^V(x, \mu) &= 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^V(\mu) C_n^{3/2}(2x-1) \right], \\ \Phi_{\perp}^V(x, \mu) &= 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^{\perp, V}(\mu) C_n^{3/2}(2x-1) \right],\end{aligned}\tag{18}$$

and twist-3 ones

$$\begin{aligned}\Phi_p(x) &= 1, \quad \Phi_{\sigma}(x) = 6x(1-x), \\ \Phi_v(x, \mu) &= 3 \left[2x-1 + \sum_{n=1}^{\infty} a_n^{\perp, V}(\mu) P_{n+1}(2x-1) \right],\end{aligned}\tag{19}$$

where $C_n(x)$ and $P_n(x)$ are the Gegenbauer and Legendre polynomials, respectively. $a_n(\mu)$ are Gegenbauer moments that depend on the scale μ . $\Phi_{\perp}^V(x, \mu)$ and $\Phi_{\parallel}^V(x, \mu)$ are the transverse and longitudinal quark distributions of the polarized mesons.

At order α_s a correction from penguin contractions is present only for $i = 4, 6$. For $i = 4$ we obtain

$$\begin{aligned}P_4^p(M_2) &= \frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \left[\frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - G_{M_2}(s_p) \right] \right. \\ &\quad + C_3 \left[\frac{8}{3} \ln \frac{m_b}{\mu} + \frac{4}{3} - G_{M_2}(0) - G_{M_2}(1) \right] \\ &\quad + (C_4 + C_6) \left[\frac{4n_f}{3} \ln \frac{m_b}{\mu} - (n_f - 2)G_{M_2}(0) \right. \\ &\quad \left. \left. - G_{M_2}(s_c) - G_{M_2}(1) \right] \right. \\ &\quad \left. - 2C_{8g}^{\text{eff}} \int_0^1 \frac{dx}{1-x} \Phi_{M_2}(x) \right\},\end{aligned}\tag{20}$$

where $n_f = 5$ is the number of light quark flavors, and $s_u = 0$, $s_c = (m_c/m_b)^2$ are mass ratios involved in the evaluation of the penguin diagrams. The function $G_{M_2}(s)$ is given

by

$$G_{M_2}(s) = \int_0^1 dx G(s - i\epsilon, 1 - x) \Phi_{M_2}(x), \quad (21)$$

$$\begin{aligned} G(s, x) &= -4 \int_0^1 du u(1 - u) \ln[s - u(1 - u)x] \\ &= \frac{2(12s + 5x - 3x \ln s)}{9x} \\ &\quad - \frac{4\sqrt{4s - x}(2s + x)}{3x^{3/2}} \arctan \sqrt{\frac{x}{4s - x}}. \end{aligned} \quad (22)$$

For $i = 6$, if M_2 is a vector meson, the result for the penguin contribution is

$$\begin{aligned} P_6^p(M_2) &= -\frac{C_F \alpha_s}{4\pi N_c} \left\{ C_1 \hat{G}_{M_2}(s_p) + C_3 [\hat{G}_{M_2}(0) + \hat{G}_{M_2}(1)] \right. \\ &\quad \left. + (C_4 + C_6) \left[(n_f - 2) \hat{G}_{M_2}(0) + \hat{G}_{M_2}(s_c) \right. \right. \\ &\quad \left. \left. + \hat{G}_{M_2}(1) \right] \right\}. \end{aligned} \quad (23)$$

In analogy with (21), the function $\hat{G}_{M_2}(s)$ is defined as

$$\hat{G}_{M_2}(s) = \int_0^1 dx G(s - i\epsilon, 1 - x) \Phi_{M_2}(x). \quad (24)$$

Electromagnetic corrections are present for $i = 8, 10$ and correspond to the penguin diagrams. For $i = 10$ we obtain

$$\begin{aligned} P_{10}^p(M_2) &= \frac{\alpha}{9\pi N_c} \left\{ (C_1 + N_c C_2) \left[\frac{4}{3} \ln \frac{m_b}{\mu} \right. \right. \\ &\quad \left. \left. + \frac{2}{3} - G_{M_2}(s_p) \right] - 3C_{7\gamma}^{\text{eff}} \int_0^1 \frac{dx}{1 - x} \Phi_{M_2}(x) \right\}. \end{aligned} \quad (25)$$

For $i = 8$

$$P_8^p(M_2) = -\frac{\alpha}{9\pi N_c} (C_1 + N_c C_2) \hat{G}_{M_2}(s_p), \quad (26)$$

if M_2 is a vector meson.

The correction from hard gluon exchange between M_2 and the spectator quark is given by

$$\begin{aligned} H_i(M_1 M_2) &= \frac{B_{M_1 M_2}}{A_{M_1 M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx \int_0^1 dy \left[\frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{\bar{x}\bar{y}} \right. \\ &\quad \left. + r_\chi^{M_1} \frac{\Phi_{M_2}(x) \Phi_{m_1}(y)}{x\bar{y}} \right], \end{aligned} \quad (27)$$

for $i = 1-4, 9, 10$.

$$H_i(M_1 M_2) = -\frac{B_{M_1 M_2}}{A_{M_1 M_2}} \frac{m_B}{\lambda_B} \int_0^1 dx \int_0^1 dy \left[\frac{\Phi_{M_2}(x) \Phi_{M_1}(y)}{x\bar{y}} + r_\chi^{M_1} \frac{\Phi_{M_2}(x) \Phi_{m_1}(y)}{\bar{x}\bar{y}} \right], \quad (28)$$

for $i = 5, 7$, and $H_i(M_1 M_2) = 0$ for $i = 6, 8$.

where λ_B is defined by

$$\int_0^1 \frac{d\xi}{\xi} \Phi_B(\xi) \equiv \frac{m_B}{\lambda_B} \quad (29)$$

with $\Phi_B(\xi)$ is one of the two light-cone distribution amplitudes of the B meson.

If $M_1 = P$, $M_2 = V$, f refers to decay constant of relevant meson, $A_{M_1 M_2}$ and $B_{M_1 M_2}$ are given by

$$A_{M_1 M_2} = i \frac{G_F}{\sqrt{2}} (-2) m_{M_1} \epsilon_{M_1}^* \cdot p_B F_0^{B \rightarrow M_1}(0) f_{M_2}, \quad (30)$$

$$B_{M_1 M_2} = -\frac{G_F}{\sqrt{2}} f_{B_s} f_{M_1} f_{M_2}. \quad (31)$$

where m_{M_1} and ϵ_{M_1} are the mass and polarization vector of the vector meson. $F_0^{B \rightarrow M_1}$ is the form factor for $B \rightarrow M_1$ transition.

We recall that the term involving $r_\chi^{M_1}$ is suppressed by a factor of Λ_{QCD}/m_b in heavy-quark power counting. Since the twist-3 distribution amplitude $\Phi_{m_1}(y)$ does not vanish at $y = 1$, the power-suppressed term is divergent. We extract this divergence by defining a parameter $X_H^{M_1}$ through

$$\begin{aligned} \int_0^1 \frac{dy}{\bar{y}} \Phi_{m_1}(y) &= \Phi_{m_1}(1) \int_0^1 \frac{dy}{\bar{y}} \\ &+ \int_0^1 \frac{dy}{\bar{y}} [\Phi_{m_1}(y) - \Phi_{m_1}(1)] \\ &\equiv \Phi_{m_1}(1) X_H^{M_1} + \int_0^1 \frac{dy}{[\bar{y}]_+} \Phi_{m_1}(y). \end{aligned} \quad (32)$$

The remaining integral is finite (it vanishes for pseudoscalar mesons since $\Phi_p(y) = 1$), but $X_H^{M_1}$ is an unknown parameter representing a soft-gluon interaction with the spectator quark. Since $X_H^{M_1}$ varies within a certain range (specified later) and $X_H^M \sim \ln(m_b/\Lambda_{\text{QCD}})[8]$, we treat the resulting variation of the coefficients α_i^p as an uncertainty. We also assume that $X_H^{M_1}$ is universal, i.e., that it does not depend on M_1 and on the index i of $H_i(M_1 M_2)$. For the convolution integrals, one can find the results in Ref. [8].

For the annihilation contribution, one can get[8]:

$$b_3^p = \frac{C_F}{N_c^2} [C_3 A_1^i + C_5 (A_3^i + A_3^f) + N_c C_6 A_3^f], \quad (33)$$

$$b_{3,\text{EW}}^p = \frac{C_F}{N_c^2} \left[C_9 A_1^i + C_7 (A_3^i + A_3^f) + N_c C_8 A_3^f \right]. \quad (34)$$

The weak annihilation kernels exhibit endpoint divergences, which we treat in the same manner as the power corrections to the hard spectator scattering. The divergent subtractions are interpreted as

$$\int_0^1 \frac{dy}{y} \rightarrow X_A^{M_1}, \quad \int_0^1 dy \frac{\ln y}{y} \rightarrow -\frac{1}{2} (X_A^{M_1})^2, \quad (35)$$

and similarly for M_2 with $y \rightarrow \bar{x}$. The treatment of weak annihilation is model-dependent in the QCD factorization approach. We treat X_A^M as an unknown complex number of order $\ln(m_b/\Lambda_{\text{QCD}})$ and make the simplifying assumption that this number is independent of the identity of the meson M_1 and the weak decay vertex. Here,

$$\begin{aligned} A_1^i \approx -A_2^i \approx 6\pi\alpha_s \left[3 \left(X_A - 4 + \frac{\pi^2}{3} \right) \right. \\ \left. + r_\chi^{M_1} r_\chi^{M_2} (X_A^2 - 2X_A) \right], \end{aligned} \quad (36)$$

$$\begin{aligned} A_3^i \approx 6\pi\alpha_s \left[-3r_\chi^{M_2} \left(X_A^2 - 2X_A - \frac{\pi^2}{3} \right) \right. \\ \left. + 4 \right] + r_\chi^{M_1} \left(X_A^2 - 2X_A + \frac{\pi^2}{3} \right), \end{aligned} \quad (37)$$

$$\begin{aligned} A_3^f \approx -6\pi\alpha_s \left[3r_\chi^{M_2} (2X_A - 1)(2 - X_A) \right. \\ \left. - r_\chi^{M_1} (2X_A^2 - X_A) \right] \end{aligned} \quad (38)$$

and $A_1^f = A_2^f = 0$. Here, M_1 is K^0 meson and M_2 is ρ^0 meson.

III. CP VIOLATION IN $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$ DECAY

A. Formalism

In the vector meson dominance model [12], the photon propagator is dressed by coupling to vector mesons. Based on the same mechanism, $\rho - \omega$ mixing was proposed [13]. The formalism for CP violation in the decay of a bottom hadron, B_s , will be reviewed in the following. The amplitude for $B_s \rightarrow K^0 \pi^+ \pi^-$, A , can be written as

$$A = \langle \pi^+ \pi^- K^0 | H^T | \bar{B}_s \rangle + \langle \pi^+ \pi^- K^0 | H^P | \bar{B}_s \rangle, \quad (39)$$

where H^T and H^P are the Hamiltonians for the tree and penguin operators, respectively. We define the relative magnitude and phases between these two contributions as follows:

$$A = \langle \pi^+ \pi^- K^0 | H^T | \bar{B}_s \rangle [1 + r e^{i\delta} e^{i\phi}], \quad (40)$$

where δ and ϕ are strong and weak phase differences, respectively. The weak phase difference ϕ arises from the appropriate combination of the CKM matrix elements: $\phi = \arg[(V_{tb}V_{ts}^*)/(V_{ub}V_{us}^*)]$. The parameter r is the absolute value of the ratio of tree and penguin amplitudes,

$$r = \left| \frac{\langle \pi^+ \pi^- K^0 | H^P | \bar{B}_s \rangle}{\langle \pi^+ \pi^- K^0 | H^T | \bar{B}_s \rangle} \right|. \quad (41)$$

The amplitude for $B_s \rightarrow \bar{K}^0 \pi^+ \pi^-$ is

$$\bar{A} = \langle \pi^+ \pi^- \bar{K}^0 | H^T | B_s \rangle + \langle \pi^+ \pi^- \bar{K}^0 | H^P | B_s \rangle. \quad (42)$$

Then, the CP violating asymmetry, a , can be written as

$$a = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r \sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2}. \quad (43)$$

We can see explicitly from Eq. (40) that both weak and strong phase differences are needed to produce CP violation. $\rho - \omega$ mixing has the dual advantages that the strong phase difference is large and well known [3, 4]. In this scenario one has

$$\langle \pi^+ \pi^- K^0 | H^T | \bar{B}_s \rangle = \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega}(t_\omega + t_\omega^a) + \frac{g_\rho}{s_\rho} (t_\rho + t_\rho^a), \quad (44)$$

$$\langle \pi^+ \pi^- K^0 | H^P | \bar{B}_s \rangle = \frac{g_\rho}{s_\rho s_\omega} \tilde{\Pi}_{\rho\omega}(p_\omega + p_\omega^a) + \frac{g_\rho}{s_\rho} (p_\rho + p_\rho^a), \quad (45)$$

where t_V ($V = \rho$ or ω) is the tree amplitude and p_V is the penguin amplitude for producing a vector meson, V . t_V^a ($V = \rho$ or ω) is the tree annihilation amplitude and p_V^a is the penguin annihilation amplitude. g_ρ is the coupling for $\rho^0 \rightarrow \pi^+ \pi^-$, $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude, and s_V is from the inverse propagator of the vector meson V ,

$$s_V = s - m_V^2 + im_V \Gamma_V, \quad (46)$$

with \sqrt{s} being the invariant mass of the $\pi^+ \pi^-$ pair.

The direct $\omega \rightarrow \pi^+ \pi^-$ is effectively absorbed into $\tilde{\Pi}_{\rho\omega}$, leading to the explicit s dependence of $\tilde{\Pi}_{\rho\omega}$ [14]. Making the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega^2) \tilde{\Pi}'_{\rho\omega}(m_\omega^2)$, the $\rho - \omega$ mixing parameters were determined in the fit of Gardner and O'Connell [15]: $\text{Re} \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -3500 \pm 300 \text{ MeV}^2$, $\text{Im} \tilde{\Pi}_{\rho\omega}(m_\omega^2) = -300 \pm 300 \text{ MeV}^2$, and $\tilde{\Pi}'_{\rho\omega}(m_\omega^2) = 0.03 \pm 0.04$. In practice, the effect of the derivative term is negligible. From Eqs. (40)(41)(44)(45) one has

$$re^{i\delta} e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega}(p_\omega + p_\omega^a) + s_\omega(p_\rho + p_\rho^a)}{\tilde{\Pi}_{\rho\omega}(t_\omega + t_\omega^a) + s_\omega(t_\rho + t_\rho^a)}. \quad (47)$$

Defining

$$\frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = r' e^{i(\delta_q + \phi)}, \quad \frac{t_\omega + t_\omega^a}{t_\rho + t_\rho^a} = \alpha e^{i\delta_\alpha}, \quad \frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \beta e^{i\delta_\beta}, \quad (48)$$

where δ_α , δ_β , and δ_q are strong phases, one finds the following expression from Eq. (47):

$$r e^{i\delta} = r' e^{i\delta_q} \frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{s_\omega + \tilde{\Pi}_{\rho\omega} \alpha e^{i\delta_\alpha}}. \quad (49)$$

$\alpha e^{i\delta_\alpha}$, $\beta e^{i\delta_\beta}$, and $r e^{i\delta}$ will be calculated in the QCD factorization approach later. With Eq. (49), we can obtain $r \sin \delta$ and $r \cos \delta$. In order to get the CP violating asymmetry, a , in Eq. (43), $\sin \phi$ and $\cos \phi$ are needed. ϕ is determined by the CKM matrix elements. In the Wolfenstein parametrization [16], one has

$$\sin \phi = \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}, \quad (50)$$

$$\cos \phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}. \quad (51)$$

B. CP violation via $\rho - \omega$ mixing

In the following we will study the CP violating asymmetries in the following decay: $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$. With the Eq. (4)(6)(7)(8), we can calculate the decay amplitudes in QCD factorization scheme. The expressions for the $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$ amplitudes are given by

$$\begin{aligned} \sqrt{2} A_{\bar{B}_s^0 \rightarrow K^0 \rho^0} &= A_{K^0 \rho^0} (\delta_{pu} \alpha_2 - \alpha_4^p + \frac{3}{2} \alpha_{3,EW}^p \\ &\quad + \frac{1}{2} \alpha_{4,EW}^p - \beta_3^p + \frac{1}{2} \beta_{3,EW}^p), \end{aligned} \quad (52)$$

$$\begin{aligned} \sqrt{2} A_{\bar{B}_s^0 \rightarrow K^0 \omega} &= A_{K^0 \omega} (\delta_{pu} \alpha_2 + 2\alpha_3^p + \alpha_4^p + \frac{1}{2} \alpha_{3,EW}^p \\ &\quad - \frac{1}{2} \alpha_{4,EW}^p + \beta_3^p - \frac{1}{2} \beta_{3,EW}^p), \end{aligned} \quad (53)$$

where

$$A_{K^0 \rho^0} = (-2) i \frac{G_F}{\sqrt{2}} m_{\rho^0} \varepsilon_{\rho^0}^* \cdot p_B F_0^{B_s \rightarrow K^0}(0) f_{\rho^0}, \quad (54)$$

$$A_{K^0 \omega} = (-2) i \frac{G_F}{\sqrt{2}} m_\omega \varepsilon_\omega^* \cdot p_B F_0^{B_s \rightarrow K^0}(0) f_\omega. \quad (55)$$

Here F_0 denote $B_s \rightarrow K^0$ meson form factor. m_{ρ^0} , m_ω are the mass of ρ^0 and ω mesons. $\varepsilon_{\rho^0}^*$, ε_ω^* correspond to polarizing vectors. f refers to the decay constant. Then we can get

$$\begin{aligned}\sqrt{2}A_{\bar{B}_s \rightarrow K^0 \rho^0} &= A_{K^0 \rho^0} \left[\delta_{pu} a_{2, K^0 \rho^0} - a_{4, K^0 \rho^0}^p - \gamma_\chi^{k^0} a_{6, K^0 \rho^0}^p \right. \\ &\quad + \frac{3}{2}(a_{9, K^0 \rho^0}^p + a_{7, K^0 \rho^0}^p) + \frac{1}{2}(a_{10, K^0 \rho^0} \\ &\quad \left. + \gamma_\chi^{k^0} a_{8, K^0 \rho^0}^p) - \beta_3^p + \frac{1}{2}\beta_{3, EW}^p \right],\end{aligned}\tag{56}$$

$$\begin{aligned}\sqrt{2}A_{\bar{B}_s \rightarrow K^0 \omega} &= A_{K^0 \omega} \left[\delta_{pu} a_{2, K^0 \omega} + 2(a_{3, K^0 \omega}^p + a_{5, K^0 \omega}^p) \right. \\ &\quad + a_{4, K^0 \omega}^p + \gamma_\chi^\omega a_{6, K^0 \omega}^p + \frac{1}{2}(a_{7, K^0 \omega} + a_{9, K^0 \omega}) \\ &\quad \left. - \frac{1}{2}(a_{10, K^0 \omega} + \gamma_\chi^\omega a_{8, K^0 \omega}^p) + \beta_3^p - \frac{1}{2}\beta_{3, EW}^p \right].\end{aligned}\tag{57}$$

where the form of the coefficients a_i^p at next-to-leading order in α_s is given by Eq.(12), which M_1 is K^0 meson and M_2 is ρ^0 meson. β_i is the weak annihilation contribution in QCD factorization. γ_χ is chirally-enhanced terms which we have denoted above.

From Eq. (6)(7)(48), one can get

$$\alpha e^{i\delta_\alpha} = \frac{t_\omega + t_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_1}{Q_2}.\tag{58}$$

$$\begin{aligned}Q_1 &= A_{K^0 \omega} \left\{ \delta_{pu} a_{2, K^0 \omega} + 2(a_{3, K^0 \omega}^u - a_{3, K^0 \omega}^c) \right. \\ &\quad + a_{5, K^0 \omega}^u - a_{5, K^0 \omega}^c + (a_{4, K^0 \omega}^u - a_{4, K^0 \omega}^c) \\ &\quad + \gamma_\chi^\omega (a_{6, K^0 \omega}^u - a_{6, K^0 \omega}^c) + \frac{1}{2}(a_{7, K^0 \omega}^u - a_{7, K^0 \omega}^c) \\ &\quad + a_{9, K^0 \omega}^u - a_{9, K^0 \omega}^c - \frac{1}{2} \left[a_{10, K^0 \omega}^u - a_{10, K^0 \omega}^c \right. \\ &\quad \left. \left. + \gamma_\chi^\omega (a_{8, K^0 \omega}^u - a_{8, K^0 \omega}^c) \right] \right\}\end{aligned}\tag{59}$$

$$\begin{aligned}
Q_2 = & A_{K^0\rho^0} \left\{ \delta_{pu} a_{2,K^0\rho^0} - (a_{4,K^0\rho^0}^u - a_{4,K^0\rho^0}^c) \right. \\
& - \gamma_\chi^{k_0} (a_{6,K^0\rho^0}^u - a_{6,K^0\rho^0}^c) + \frac{3}{2} (a_{9,K^0\rho^0}^u - a_{9,K^0\rho^0}^c) \\
& + a_{7,K^0\rho^0}^u - a_{7,K^0\rho^0}^c + \frac{1}{2} \left[a_{10,K^0\rho^0}^u - a_{10,K^0\rho^0}^c \right. \\
& \left. \left. + \gamma_\chi^{k_0} (a_{8,K^0\rho^0}^u - a_{8,K^0\rho^0}^c) \right] \right\}, \tag{60}
\end{aligned}$$

In a similar way, with the aid of the Fierz identities, we can evaluate the penguin operator contributions p_ρ and p_ω . From Eq. (48) we have

$$\beta e^{i\delta_\beta} = \frac{p_\rho + p_\rho^a}{p_\omega + p_\omega^a} = \frac{Q_3}{Q_4}, \tag{61}$$

where

$$\begin{aligned}
Q_3 = & A_{K^0\rho^0} \left[-a_{4,K^0\rho^0}^c - \gamma_x^{k_0} a_{6,K^0\rho^0}^c \right. \\
& + \frac{3}{2} (a_{9,K^0\rho^0}^c + a_{7,K^0\rho^0}^c) + \frac{1}{2} (a_{10,K^0\rho^0}^c \\
& \left. + \gamma_x^{k_0} a_{8,K^0\rho^0}^c) - \beta_3 + \frac{1}{2} \beta_{3,EW} \right], \tag{62}
\end{aligned}$$

$$\begin{aligned}
Q_4 = & A_{K^0\omega} \left[2(a_{3,K^0\omega}^c + a_{5,K^0\omega}^c) \right. \\
& + a_{4,K^0\omega}^c + \gamma_x^\omega a_{6,K^0\omega}^c + \frac{1}{2} (a_{7,K^0\omega}^c + a_{9,K^0\omega}^c) \\
& \left. - \frac{1}{2} (a_{10,K^0\omega}^c + \gamma_x^\omega a_{8,K^0\omega}^c) + \beta_3 - \frac{1}{2} \beta_{3,EW} \right]. \tag{63}
\end{aligned}$$

and

$$r' e^{i(\delta_q + \phi)} = \frac{p_\omega + p_\omega^a}{t_\rho + t_\rho^a} = \frac{Q_4}{Q_2}, \tag{64}$$

$$r' e^{i\delta_q} = \frac{Q_4}{Q_2} \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right|, \tag{65}$$

where

$$\left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right| = \frac{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}{(1 - \lambda^2/2)(\rho^2 + \eta^2)}. \tag{66}$$

It can be seen that r' and δ_q depend on both the Wilson coefficients and the CKM matrix elements, as shown in Eqs. (65). Substituting Eqs. (58) (61) (65) into Eq. (49), we can obtain r , $\sin \delta$, and $\cos \delta$. Then, in combination with Eqs. (50) and (51) the CP violating asymmetries can be obtained.

IV. BRANCHING RATIO OF $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$

The matrix element for $B_s \rightarrow P$ and $B_s \rightarrow V$ (where P and V denote pseudoscalar and vector mesons, respectively) can be decomposed as follows [17]:

$$\begin{aligned}
\langle P | J_\mu | B_s \rangle &= \left(p_{B_s} + p_P - \frac{m_{B_s}^2 - m_P^2}{k^2} k \right)_\mu F_1(k^2) \\
&\quad + \frac{m_{B_s}^2 - m_P^2}{k^2} k_\mu F_0(k^2), \\
\langle V | J_\mu | B_s \rangle &= \frac{2}{m_{B_s} + m_V} \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p_{B_s}^\rho p_V^\sigma V(k^2) \\
&\quad + i \left\{ \epsilon_\mu^* (m_{B_s} + m_V) A_1(k^2) - \frac{\epsilon^* \cdot k}{m_{B_s} + m_V} \right. \\
&\quad \times (p_{B_s} + p_V)_\mu A_2(k^2) - \frac{\epsilon^* \cdot k}{k^2} 2m_V \cdot k_\mu A_3(k^2) \Big\} \\
&\quad + i \frac{\epsilon^* \cdot k}{k^2} 2m_V \cdot k_\mu A_0(k^2),
\end{aligned} \tag{67}$$

where J_μ is the weak current ($J_\mu = \bar{q} \gamma^\mu (1 - \gamma_5) b$ with $q = u, d, s$), $p_{B_s}(m_{B_s}), p_P(m_P), p_V(m_V)$ are the momenta (masses) of B_s, P, V , respectively, $k = p_{B_s} - p_P(p_V)$ for $B_s \rightarrow P(V)$ transition and ϵ_μ is the polarization vector of V . F_i ($i = 0, 1$) and A_i ($i = 0, 1, 2, 3$) in Eq. (67) are the weak form factors which satisfy $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$, and $A_3(k^2) = [(m_B + m_V)/2m_V] A_1(k^2) - [(m_B - m_V)/2m_V] A_2(k^2)$.

With the factorizable decay amplitudes in Eq. (56)(57) we can calculate the decay rate for B_s to a pseudoscalar meson (P) and a vector meson (V) transition by using the following expression [18]:

$$\Gamma(B_s \rightarrow PV) = \frac{p_c}{8\pi m_V^2} |A(B_s \rightarrow PV)/(\epsilon \cdot p_{B_s})|^2, \tag{68}$$

where

$$p_c = \frac{\sqrt{[m_{B_s}^2 - (m_P + m_V)^2][m_{B_s}^2 - (m_P - m_V)^2]}}{2m_{B_s}}$$

is the c.m. momentum of the product particle and $A(B_s \rightarrow PV)$ is the decay amplitude.

In the QCD factorization approach. Here $V_u^{T,P}$ are the CKM factors,

$$V_u^T = |V_{ub} V_{uq}^*|, \text{ for } i = 1, 2, \tag{69}$$

and

$$V_u^P = |V_{tb}V_{tq}^*|, \text{ for } i = 3, \dots, 10. \quad (70)$$

In our case we take into account the $\rho - \omega$ mixing contribution when we calculate the branching ratios since we are working to the first order of isospin violation. we can explicitly express the branching ratio for the process $\bar{B}_s \rightarrow K^0 \rho^0(\omega)$ as the following:

$$\begin{aligned} & BR(\bar{B}_s \rightarrow K^0 \rho^0(\omega)) \\ &= \frac{G_F^2 p_c^3}{16\pi m_\rho^2 \Gamma_{B_s}} |[V_u^T A_{\rho^0}^T(a_1, a_2) - V_u^P A_{\rho^0}^P(a_3, \dots, a_{10})] \\ &+ [V_u^T A_\omega^T(a_1, a_2) - V_u^P A_\omega^P(a_3, \dots, a_{10})] \\ &\times \frac{\tilde{\Pi}_{\rho\omega}}{(s_\rho - m_\omega^2) + im_\omega \Gamma_\omega}|^2, \end{aligned} \quad (71)$$

where Γ_{B_s} is the total decay width of B_s .

V. INPUT PARAMETERS

In the numerical calculations, we have several parameters, i.e. N_c and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which should be determined from experiments, we use the results of Ref. [2]:

$$\begin{aligned} \bar{\rho} &= 0.132_{-0.014}^{+0.022}, \quad \bar{\eta} = 0.341 \pm 0.013, \\ \lambda &= 0.2253 \pm 0.0007, \quad A = 0.808_{-0.015}^{+0.022}. \end{aligned} \quad (72)$$

In QCD factorization scheme, since power corrections have been considered, N_c is only color parameter, hence we use $N_c = 3$. In naive factorization N_c includes the nonfactorizable effects which are model and process dependent and cannot be theoretically evaluated accurately and can be determined by experiment.

The running quark masses is taken by the scale μ in B_s decay. One has

$$\begin{aligned} m_b(m_b) &= 4.2 \text{ GeV}, \quad m_c(m_b) = 0.91 \text{ GeV}, \\ m_u(m_b) &= m_d(m_b) = 0, \quad m_s(2.1 \text{ GeV}) = 0.095 \text{ GeV}. \end{aligned} \quad (73)$$

The values of the scale dependent quantities $f_V^\perp(\mu)$ and $a_{1,2}^\perp(\mu)$ are given for $\mu = 1 \text{ GeV}$. The value of Gegenbauer moments are taken from [19].

$$\begin{aligned} a_1^\rho &= 0, \quad a_2^\rho = 0.15 \pm 0.07 \\ a_1^\omega &= 0, \quad a_2^\omega = 0.15 \pm 0.07 \\ a_1^{\perp\rho} &= 0, \quad a_2^{\perp\rho} = 0.14 \pm 0.06 \\ a_1^{\perp\omega} &= 0, \quad a_2^{\perp\omega} = 0.14 \pm 0.06 \\ a_1^K &= 0.06 \pm 0.03, \quad a_2^K = 0.25 \pm 0.15 \\ f_\rho &= 216 \pm 3 \text{ MeV}, \quad f_\rho^\perp(\mu) = 165 \pm 9 \text{ MeV}, \\ f_\omega &= 187 \pm 5 \text{ MeV}, \quad f_\omega^\perp(\mu) = 151 \pm 9 \text{ MeV}, \end{aligned} \quad (74)$$

For B_s meson, we use the value[2]:

$$\tau = 1.47ps, \quad m_{B_s} = 5.366GeV \quad (75)$$

The Wilson coefficients c_i can be found in [8]. As discussed in detail in [8], there are large theoretical uncertainties related to the modeling of power corrections corresponding to weak annihilation effects and the chirally-enhanced power corrections to hard spectator scattering. So we parameterize these effects in terms of the divergent integrals X_H (hard spectator scattering) and X_A (weak annihilation). We also model these quantities by using the parameterization[8]

$$X_A = \left(1 + \varrho_A e^{i\varphi_A}\right) \ln \frac{m_B}{\Lambda_h}; \quad \varrho_A \leq 1 \quad \Lambda_h = 0.5 \text{ GeV}, \quad (76)$$

and similarly for X_H . Here φ_A is an arbitrary strong-interaction phase, which may be caused by soft rescattering. The fitted ϱ_A and φ_A are taken from [20]. For $B_s \rightarrow PV$ decay, $\rho_A^{PV} \approx 0.87$, $\phi_A^{PV} \approx -30^\circ$. For the estimate of theoretical uncertainties, we shall assign an error of ± 0.1 to ρ_A and $\pm 20^\circ$ to ϕ_A [20].

The form factors associated with the weak transitions depend on the inner structure of the hadrons and are hence model dependent. Here we will consider the form factors obtained in several phenomenological models. For B_s decay form factors, we will use the results (form factors are referred to the ones at $q^2 = 0$):

1). Model 1 [8]

$$F_0^{B_s \rightarrow K} = 0.31 \pm 0.05,$$

2). Model 2 (in the pQCD approach)[21]

$$F_0^{B_s \rightarrow K} = 0.24_{-0.04-0.01}^{+0.05+0.00},$$

3). Model 3 (form factors obtained by QCD sum rules)[22]

$$F_0^{B_s \rightarrow K} = 0.30_{-0.03}^{+0.04},$$

4). Model 4 (light-cone sum rule calculation based on heavy quark effective theory)[23]

$$F_0^{B_s \rightarrow K} = 0.296 \pm 0.018,$$

5). Model 5 (A light cone quark model in conjunction with soft collinear effective theory)[24]

$$F_0^{B_s \rightarrow K} = 0.290,$$

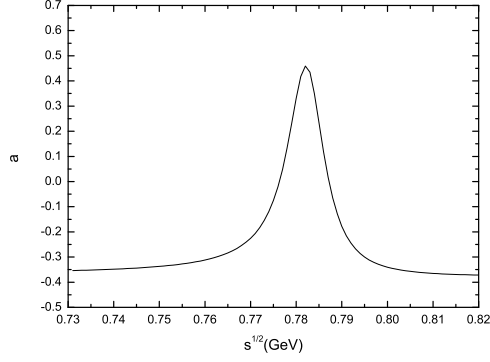


FIG. 1: Plot of a as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$.

6). Model 6 (lattice QCD calculation)[25]

$$F_0^{B_s \rightarrow K} = 0.23 \pm 0.05 \pm 0.04.$$

In above Models, the k^2 dependence of the form factors has the following form under the nearest pole dominance assumption:

$$h(k^2) = \frac{h(0)}{1 - \frac{k^2}{m_h^2}}, \quad (77)$$

where h could be F_0 , and m_h is the pole mass.

It is noted that since the value of k^2 (which is actually the square of the mass of the factorized light meson) is much smaller than the square of the pole mass which is of order m_b^2 , only the values of the form factors at $k^2 = 0$ are most relevant and hence how the form factors depend on k^2 has little effects (less than 2%). From the above values we see that the form factor $B_s \rightarrow K$ at $q^2 = 0$ ranges from 0.23 to 0.31.

VI. NUMERICAL RESULTS AND DISCUSSIONS

A. CP violation via $\rho - \omega$ mixing in $\bar{B}_s^0 \rightarrow K^0 \pi^+ \pi^-$

In the numerical calculations, we find the CP violating asymmetry, a , is large when the invariant mass of $\pi^+ \pi^-$ is in the vicinity of the ω resonance within QCD factorization scheme.

In the respective error ranges, when $\sqrt{s} = 0.782 \text{ GeV}$, we get maximum CP violating asymmetry

$$a = (45.9_{-15.7}^{+16.2+27.5}) \times 10^{-2} \quad (78)$$

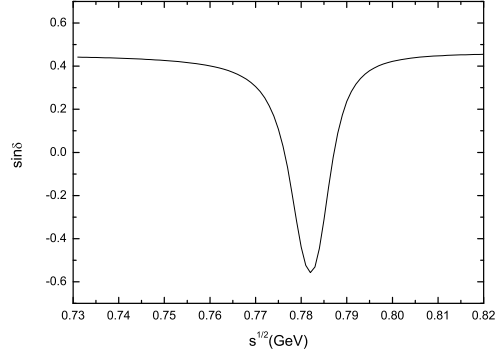


FIG. 2: Plot of $\sin \delta$ as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements with $\rho - \omega$ mixing for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$.

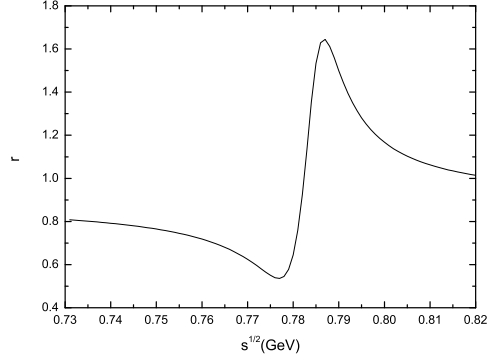


FIG. 3: Plot of r as a function of \sqrt{s} corresponding to central parameter values of CKM matrix elements with $\rho - \omega$ mixing for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$.

In QCD factorization, the theoretical errors are large which follows to the uncertainties of results. Generally, power corrections beyond the heavy quark limit give the major theoretical uncertainties. This implies the necessity of introducing $1/m_b$ power corrections. Unfortunately, there are many possible $1/m_b$ power suppressed effects and they are generally nonperturbative in nature and hence not calculable by the perturbative method. There are more uncertainties in this scheme. The first error refers to the variation of the CKM parameters. The second error comes from form factors and decay constants. The third error corresponds to the Gegenbauer moments. The last error is the wave function of the B_s meson characterized by the parameter λ_B , the power corrections due to weak annihilation and hard spectator interactions described by the parameters $\rho_{A,H}$, $\phi_{A,H}$, respectively. Using the central values of above parameters, we first calculate the numerical results of CP violation and branching ratio, and then add errors according to standard deviation. In Fig.1, We give the central value of CP violating asymmetry as a function of \sqrt{s} . From the figure one can see the CP asymmetry parameter is dependent on \sqrt{s} and changes rapidly due to $\rho - \omega$ mixing when the invariant mass of $\pi^+ \pi^-$ is in the vicinity of

the ω resonance. The CP violating asymmetry vary from around -37% to around 45% .

From Eq. (43), one can see that the CP violating asymmetry parameter depends on both $\sin \delta$ and r . The plots of $\sin \delta$ and r as a function of \sqrt{s} are shown in Fig. 2 and Fig. 3. It can be seen that when $\rho - \omega$ mixing is taken into account $\sin \delta$ and r change sharply when the invariant mass of $\pi^+\pi^-$ is around 0.782 GeV. From the Fig. 2, one can find $\rho - \omega$ mixing make the $\sin \delta$ value oscillate from -0.56 to 0.44 which can not reach the value -1 . This result is not in agreement with conclusion from naive factorization which can measure the CP violating parameter to remove the $\text{mod}(\pi)$ phase uncertainty in the determination of the CKM angle α arising from the conventional determination through $\sin 2\alpha$ [7].

We have shown that $\rho - \omega$ mixing does enhance the direct CP violating asymmetries and provide a mechanism for large CP violation in QCD factorization scheme. On the other hand, it is important to see whether it is possible to observe these large CP violating asymmetries in experiments. This depends on the branching ratio for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$. We will study this problem in the next section.

B. Branching ratios via $\rho - \omega$ mixing in $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$

Including $\rho - \omega$ mixing, we calculate the values of branching ratios for $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$. Base on the reasonable parameters range, we obtain the branching ratio of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$ is $(9.8_{-1.2}^{+2.6+3.4}) \times 10^{-7}$ which is consistent with the result [20]. In other words, although we calculate the branching ratio due to $\rho - \omega$ mixing in QCD factorization scheme, we find the contribution of $\rho - \omega$ mixing for branching ratio is small and can be neglected. $\rho - \omega$ mixing mechanism presents new phase differences and produce extremely small effect for branching ratio of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega)$.

VII. DISCUSSIONS ON POSSIBILITY TO OBSERVE CP VIOLATING ASYMMETRIES AT THE LHC

The LHC is a proton-proton collider currently have started at CERN. With the designed center-of-mass energy 14 TeV and luminosity $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, the LHC gives access to high energy frontier at TeV scale and an opportunity to further improve the consistency test for the CKM matrix. The production rates for heavy quark flavours will be large at the LHC, and the $b\bar{b}$ production cross section will be of the order 0.5 mb, providing as many as 0.5×10^{12} bottom events per year [26]. In particular, the LHCb detector is designed to exploit large number of b -hadrons produced at the LHC in order to make precise studies on CP asymmetries and on rare decays in b -hadron systems. The other two experiments, ATLAS and CMS, are optimized for discovering new physics and will complete most of their B physics program within the first few years [26, 27]. Obviously, the LHC has a great advantage over the current experiments on b -hadrons[28].

In the present work, we have predicted possible large CP violating asymmetries in decay channel of $\bar{B}_s^0 \rightarrow K^0 \rho^0(\omega) \rightarrow K^0 \pi^+ \pi^-$ via the $\rho - \omega$ mixing. At the LHC, the b -hadrons are produced from the pp collisions. The possible asymmetry between the numbers of the b -hadrons, H_b , and those of their antiparticles, \bar{H}_b , has been studied in the Lund string fragmentation model and the intrinsic heavy quark model [29, 30]. It has

been shown that this asymmetry can only reach values of a few percent. In our following discussions, we will ignore this small asymmetry and give the numbers of $H_b\bar{H}_b$ pairs needed for observing the CP violating asymmetries we have predicted. These numbers depend on both the magnitudes of the CP violating asymmetries and the branching ratios of heavy hadron decays which are model dependent. For one (three) standard deviation signature, the number of $H_b\bar{H}_b$ pairs we need is [31–33]

$$N_{H_b\bar{H}_b} \sim \frac{1}{BRa^2}(1 - a^2) \left(\frac{9}{BRa^2}(1 - a^2) \right), \quad (79)$$

where BR is the branching ratio for $H_b \rightarrow f\rho^0$.

For central value of CP asymmetry in Eq. (78), we present the numbers of $B_s\bar{B}_s$ pairs for observing the large CP violating asymmetries at LHC. For the channel $\bar{B}_s^0 \rightarrow K^0\rho^0(\omega) \rightarrow K^0\pi^+\pi^-$, the numbers of $B_s\bar{B}_s$ pairs are 3.8×10^6 (3.4×10^7) for 1σ (3σ) signature. At the LHC the average $B_s\bar{B}_s$ production is about 10% out of 10^{12} $b\bar{b}$ events [26]. From Fig.1, one can see CP violating asymmetries vary sharply at small energy range, and reach peak value at $\sqrt{s} = 0.782$ GeV. Hence, it is very possible to observe the large CP violating asymmetries in small energy range of $\rho^0 \sim \omega$ resonance at the peak values of CP violating asymmetries from the LHC experiment. For the experiments, it is possible to reconstruction π^+ , π^- and K^0 mesons when the invariant masses of $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. Therefore, it is very possible to observe the large CP violating asymmetries in $\bar{B}_s^0 \rightarrow K^0\rho^0(\omega) \rightarrow K^0\pi^+\pi^-$ at the LHC.

VIII. SUMMARY AND CONCLUSIONS

In this paper, we have studied CP violation in $\bar{B}_s^0 \rightarrow K^0\rho^0(\omega) \rightarrow K^0\pi^+\pi^-$. It has been found that, by including $\rho - \omega$ mixing, the CP violating asymmetries can be large when the invariant masses of $\pi^+\pi^-$ pairs are in the vicinity of the ω resonance. For the decay $\bar{B}_s^0 \rightarrow K^0\rho^0(\omega) \rightarrow K^0\pi^+\pi^-$, the maximum CP violation can reach 46%. Furthermore, taking $\rho - \omega$ mixing into account, we have calculated the branching ratio of the decay. We have also presented the numbers of $B_s\bar{B}_s$ pairs required for observing the predicted CP violation in experiments at the LHC. We have found the channel is the likely channel in which the large CP violating asymmetries may be observed at LHC. We expect that our predictions will provide a useful guidance for future investigations and experiments.

In our calculations there are some uncertainties. We have worked in the QCD factorization which is expected to be a reliable approach in the heavy-quark limit. In the QCD factorization scheme, $\alpha_s(m_b)$ and some $1/m_b$ (annihilation) corrections are included. In this framework, there is cancellation of the scale and renormalization scheme dependence between the Wilson coefficients and the hadronic matrix elements. However, the QCD factorization scheme suffers from endpoint singularities which are not well controlled. The CP violating asymmetry depends on the unknown parameters which are associated with such endpoint singularities. The CKM matrix elements also lead to some uncertainties in the CP violating asymmetry. Uncertainties also come from the weak form factors associated with the hadronic matrix elements. This lead to uncertain CP violating asymmetries in the QCD factorization scheme. This needs further detailed investigations.

Acknowledgements. This work was supported by the Special Grants (Project Number 2009BS028) for P.H.D from Henan University of Technology.

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